Aerodynamic Drag

One of Newton's laws of motion asserts that a body in motion will continue in a straight line at constant speed unless acted upon by an external force. Everyday experience seems to contradict this—straight line, maybe; constant speed, no way. A car with the power shut off will definitely not continue at a constant speed; it will gradually coast to a halt. Yet Newton was right; the reason the car slows is because there *is* an external force acting—a friction force that arises partly from the flexing of the tires, the churning of oil in the transmission, and a host of other minor causes, but mostly from aerodynamic drag.

This force has two principal causes. The first results from the air molecules immediately adjacent to the body surface tending to "stick" to that surface and so getting carried along with it, while molecules further away from the body flow past at "free stream" speed. The scrubbing of the slow-moving molecules against the faster moving ones causes *skin friction*. Skin friction is significant for aircraft and Land Speed Record "streamliners," but is only a minor component in the aerodynamic drag of passenger or race car shapes at their typical speeds.

By far the largest source of air drag on unstreamlined car shapes arises from pressure differences between the forward-facing and rearward-facing surfaces of the vehicle. Because of the comparatively clumsy shape (what aerodynamicists call a *bluff body*), air piles up in front—raising the pressure there—and is unable to close in smoothly at the back, causing a drop in the prevailing pressure at the rear of the car. The net effect of this push on the front and pull on the back is termed *form* (shape) *drag*.

The aerodynamic force attributable to the form drag, expressed in pounds, is given by:

$$F_{drag} \cong \frac{C_d \cdot A \cdot V^2}{400}$$

where: F_{drag} = aerodynamic drag, lb
C_d = drag coefficient (no units)
A = frontal area, sq ft
V = velocity, mph

We should explain some of these terms. The drag coefficient, C_d, is simply a number (with no units attached) that compares the "slipperiness" of a shape to that of a flat plate square on to the airstream. That "barn door" reference figure is taken to be 1.2; a typical modern sedan has a C_d of 0.35–0.45; a perfect "streamlined" teardrop shape might have a C_d as low as 0.03.

The frontal area, A, is simply the size of the car (or other object) viewed head-on from the front. While there are various methods for assessing a car's frontal area, ranging from tracing a photo onto graph paper and counting squares, through very precise laser "planimetry," a rough ballpark figure for any conventional automobile is 80 percent of the height times the width.

Let's take as an example a vehicle with a C_d of 0.44 and a frontal area, A, of 19.25 square feet (which, by the way, happen to be the relevant figures for a VW Beetle) running at 55 mph:

$$F_{drag} \cong \frac{C_d \cdot A \cdot V^2}{400}$$

$$\cong \frac{0.44 \times 19.25 \times 55^2}{400}$$

$$\cong 64 \text{ lb}$$

It is often more convenient to think of drag in terms of horsepower, rather than a certain number of pounds. The equation for calculating aerodynamic drag expressed as horsepower is given by:

$$Drag_{hp} \cong \frac{C_d \cdot A \cdot V^3}{15 \times 10^4}$$

where: $Drag_{hp}$ = aerodynamic drag, horsepower
C_d = drag coefficient (no units)
A = frontal area, sq ft
V = velocity, mph

We'll rework the VW Beetle example, using this equation:

$$Drag_{hp} \cong \frac{C_d \cdot A \cdot V^3}{15 \times 10^4}$$

$$\cong \frac{0.44 \times 19.25 \times 55^3}{15 \times 10^4}$$

$$\cong 9.4 \text{ hp}$$

This may seem too low, but remember that this ignores the contribution of tire drag. In truth, the real power consumption of a car running on flat ground at a modest constant speed is remarkably low. But look what happens when we increase the speed. We will pick 72 mph, because that is pretty close to the top speed of an old Beetle.

$$Drag_{hp} \cong \frac{C_d \cdot A \cdot V^3}{15 \times 10^4}$$

$$\cong \frac{0.44 \times 19.25 \times 72^3}{15 \times 10^4}$$

$$\cong 21 \text{ hp}$$

While the fact that the drag horsepower has more than doubled for just a 30 percent increase in speed drives home the point that drag horsepower varies as the *cube* of the speed (V^3), the total again seems low, but recall that we have not taken tire drag into account, which is known to be about 4–5 horsepower at that speed. We also happen to know that, while the power output of an early VW engine was 36 horsepower, measured *at the flywheel*, only 31–32 horsepower made it to the rear wheels. So, total drag at 72 mph is at least 25 horsepower, while there is 31–32 horsepower available. With just 5 or so horsepower left over, and thus available to accelerate the car, it is easy to see why these old hair dryers just ran out of steam at a bit over 70 mph.

We can take another opportunity to practice transposing terms by using the drag equation to calculate the theoretical top speed of a car when the horsepower, C_d, and A are known. Let's consider something that has about 21 square feet of frontal area, a C_d of about 0.31, and maybe 700 horsepower, figures typical of a Winston Cup car of a handful of years back.

$$Drag_{hp} \cong \frac{C_d \cdot A \cdot V^3}{15 \times 10^4}$$

$$700 = \frac{0.31 \times 21 \times V^3}{15 \times 10^4} \text{ (set } Drag_{hp} = \text{known power)}$$

$$700 \times 15 \times 10^4 = 0.31 \times 21 \times V^3 \text{ (multiply both sides by } 15 \times 10^4\text{)}$$

$$0.31 \times 21 \times V^3 = 700 \times 15 \times 10^4$$

(swap sides, to get V^3 on the left)

$$V^3 = \frac{700 \times 15 \times 10^4}{0.31 \times 21} \text{ (divide both sides by } 0.31 \times 21\text{)}$$

$$V^3 \cong 16{,}129{,}032 \text{ (do the arithmetic)}$$

$$V \cong \sqrt[3]{16{,}129{,}032}$$

$$\cong 253 \text{ mph}$$